Speakers, Titles, and Abstracts

Simone Cecchini, Texas A&M University
Metric inequalities with positive scalar curvature

We will discuss various situations where a certain perturbation of the Dirac operator on spin manifolds can be used to obtain distance estimates from lower scalar curvature bounds. A first situation consists in an area non-decreasing map from a Riemannian spin manifold with boundary X into the round sphere under the condition that the map is locally constant near the boundary and has nonzero degree. Here a positive lower bound of the scalar curvature is quantitatively related to the distance from the support of the differential of f and the boundary of X. A second situation consists in estimating the distance between the boundary components of Riemannian “bands” $M \times [-1,1]$ where M is a closed manifold that does not carry positive scalar curvature. Both situations originated from questions asked by Gromov. In the final part, I will compare the Dirac method with the minimal hypersurface method and show that if N is a closed manifold such that the cylinder N x R carries a complete metric of positive scalar curvature, then N also carries a metric of positive scalar curvature. This answers a question asked by Rosenberg and Stolz.

This talk is based on joint work with Daniel Räde and Rudolf Zeidler.

Dawei Chen, Boston College
Counting geodesics on flat surfaces

A holomorphic differential induces a flat metric with saddle points such that the underlying Riemann surface can be realized as a polygon whose edges are pairwise identified by translation. Varying such flat surfaces by affine transformations induces an action on moduli spaces of differentials, called Teichmueller dynamics. Generic flat surfaces in an orbit closure of Teichmueller dynamics possess similar properties from the viewpoint of counting geodesics of bounded lengths, whose asymptotic growth rates satisfy a formula of Siegel--Veech type. In this talk I will give an introduction to this topic and discuss some recent results about computing Siegel-Veech constants via intersection theory on moduli spaces.

Anna Fino, Florida International University
Canonical Metrics in Complex Geometry

A natural and important question in complex geometry is finding canonical Riemannian metrics that have special curvature properties. If the complex dimension is one, an answer is provided by the Uniformization Theorem, which says that any Riemann surface can be deformed in a conformal way (i.e., preserving angles) into a Riemann surface of constant curvature. The theorem was proved in 1907 and it can be seen as the one-dimensional answer to the problem in complex geometry, going back to the 1930s, asking to determine which complex manifolds admit Kähler-Einstein metrics. More broadly, in the 1950s Calabi asked whether a compact complex manifold admits a preferred Kähler metric, distinguished by natural conditions on the volume or the Ricci tensor. There has been much interest recently in extending Calabi's Programme to the case of compact complex manifolds which do not admit a Kähler metric, but rather possess "canonical metrics", where the word canonical refers to certain features of the associated fundamental form. In particular, a Hermitian metric on a complex manifold is called pluriclosed if the torsion of the associated Bismut connection is closed, and it is called balanced if its fundamental form is co-closed. In this talk I will survey recent results on pluriclosed and balanced metrics, showing new constructions of compact non-Kähler manifolds and some open problems.

David Fisher, Rice University
Hidden applications of hidden symmetries

A manifold M is said to have "hidden symmetry" if there is an isometry of a finite cover that is neither a deck transformation nor the lift of an isometry of M. Hidden symmetries can also be described in terms of commensurators of the fundamental group inside the isometry group of the universal cover.

Speculation going back to Greenberg in the 1970s indicate that infinite volume hyperbolic manifolds should not have hidden symmetries. This question was raised again and significantly generalized by Shalom for different reasons in the last 15 years. I will describe a number of surprising connections between this question and other open problems in geometry and topology and also describe current progress on the question. This is joint work with Nic Brody, Mahan Mj, and Wouter van Limbeek.

Mohammad Ghomi, Georgia Institute of Technology
Geometric Inequalities in Spaces of Nonpositive Curvature (Parts I and II)

The oldest and best known geometric inequality in mathematical analysis is the isoperimetic inequality, which states that among all regions with a given perimeter in Euclidean space balls enclose the most volume. Generalizing this fact to spaces of nonpositive curvature has been an outstanding problem in Riemannian Geometry. We will give an introduction to this problem, which is known as the Cartan-Hadamard conjecture, and discuss a number of related results and methods developed in recent years. Central to these investigations are variational techniques which lead naturally to study of total mean curvatures, or integrals of symmetric functions of the principal curvatures of hypersurfaces. These quantities appear in Steiner's formula, Brunn-Minkowski theory, and Alexandrov-Fenchel inequalities. We will describe a number of new inequalities for these integrals in nonpositively curved spaces, which are obtained via Reilly's identities, Chern's formulas, and harmonic mean curvature flow. As applications we obtain several new isoperimetric inequalities, and Riemannian rigidity theorems. This is joint work with Joel Spruck.

I plan to start discussing this topic on Friday, and continue on Saturday, although on Saturday I will give a quick summary for people who might have missed the first lecture.

Svetlana Jitomirskaya, **Georgia Institute of Technology**
Fractal properties of the Hofstadter butterfly, eigenvalues, and topological phase transitions

We will give a brief introduction to the spectral theory of ergodic operators. Then we will discuss several remarkable spectral phenomena present in the class of quasiperiodic operators and illustrate using the almost Mathieu (aka Harper's) operator - a model behind the Hofstadter's butterfly and Thouless theory of the Quantum Hall Effect. We will discuss the fascinating history of this model, that is now heavily studied in physics, and then will describe several recent results that resolve some long-standing conjectures.

Tracy Payne, **Idaho State University**
Lie Sphere Geometry and Generalized Voronoi Diagrams

The classical Voronoi diagram for a set S of points in the Euclidean plane is the subdivision of the plane into Voronoi cells, one for each point in the set. The Voronoi cell for a point p is the set of points in the plane that have p as the closest point in S. This notion is so fundamental that it arises in a multitude of contexts, both in theoretical mathematics and in the real world. The notion of Voronoi diagram may be expanded by changing the underlying geometry, by allowing the sites to be sets rather than points, by weighting sites, by subdividing the domain based on farthest point rather than closest point, or by subdividing the domain based on which k sites are closest.

Brown, Aurenhammer, Edelsbrunner and others have used projective geometry to encode various types of generalized Voronoi diagrams as ``lifting'' problems wherein the diagram in R^d is computed by translating quadratic defining conditions in R^d to linear conditions in one dimension higher, so that the diagram can be obtained by projecting the facets of a polyhedron in R^{d+1} to R^d to get the cells of the diagram.

Lie sphere geometry is the geometric study of the moduli space of Lie spheres: points (including a point at infinity), oriented spheres, and oriented hyperplanes. These are parametrized by a quadric hypersurface – the Lie quadric— in a projective space, and this space comes equipped with a natural symmetric bilinear form and a Lie group.

We use Lie sphere geometry to describe two large classes of generalized Voronoi diagrams that can be encoded as lifting problems in terms of the Lie quadric, the Lie inner product, and polyhedra. The first class includes diagrams defined in terms of extremal spheres in the space of empty spheres, and the second class includes minimization diagrams for functions that can be transformed into restrictions of affine functions to quadric hypersurfaces. These results unify and generalize previously known descriptions of generalized Voronoi diagrams as lifting problems. Special cases include classical Voronoi diagrams, power diagrams, order k and farthest point diagrams, Apollonius diagrams, medial axes, and generalized Voronoi diagrams whose sites are combinations of points, spheres and half-spaces.

Most of the talk will be an overview of Lie sphere geometry and the sharing of images of various kinds of generalized Voronoi diagrams.

Craig Sutton, **Dartmouth College**
Generic properties of eigenfunctions in the presence of torus actions

Let T be a non-trivial torus acting freely on a closed manifold M, where dim M > dim T, and let Δ_g denote the Laplace operator associated to a Riemannian metric g on \$M\$. We demonstrate that, for any integer $k \geq 2$, a generic T-invariant C^k Riemannian metric g on M has the following properties: (1) the real Δ_g eigenspaces are irreducible representations of T and, consequently, are of dimension one or two, and (2) the nodal set of any Δ_g eigenfunction is a smooth hypersurface. This is a natural generalization of a seminal result of Uhlenbeck (1976) and provides a mathematically rigorous instance of the belief in quantum mechanics that non-irreducible eigenspaces are "accidental degeneracies." Furthermore, we show that if the non-trivial quotient B = M/T is of dimension at least two and satisfies a certain topological condition, then (modulo a subspace of Weyl density zero) the nodal set of each Δ_g-eigenfunction is a smooth hypersurface dividing M into exactly two nodal domains, the minimal possible number of nodal domains for a non-constant eigenfunction. This stands in stark contrast to the observed behavior of the nodal domain count in the presence of an ergodic geodesic flow (on a surface), where the known examples suggest the nodal domain count associated to a "typical" sequence of orthogonal Laplace eigenfunctions will approach infinity.

This is joint work with Donato Cianci (GEICO), Chris Judge (Indiana) and Samuel Lin (Oklahoma).