TCU Math Newsletter

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One reason why mathematics enjoys special esteem, above all other sciences, is that its laws are absolutely certain and indisputable, while those of other sciences are to some extent debatable and in constant danger of being overthrown by newly discovered facts.

- Albert Einstein

NSF Research Experience for Undergraduates Summer 2022 Programs

The National Science Foundation (NSF) funds summer research opportunities for mathematics undergraduate students through REU Sites across the country. Students are granted stipends and, in most cases, housing and a travel allowance. The application deadlines vary, but some are in February 2022.

A list of Mathematics REU sites where you can find details and learn about the individual programs and the application processes can be found at www.nsf.gov/crssprgm/reu/list_result.jsp?unitid=5044

Another REU program that mathematics students may be interested in is the NSF REU in Data Science at Worcester Polytechnic Institute. Information about this REU can be found at www.wpi.edu/academics/departments/data-science/reu-program

TCU Math Club Meeting on February 2

The TCU Math Club will meet on Wednesday, February at 7:00 pm in TUC 243. We will be starting things with the traditional mini math competition! So bring your friends and come partake in fun small-group-based competition.

The link to join the Math GroupMe: https://groupme.com/join_group/66148722/2UpCAMFY

Texas/Oklahoma Research Undergraduate Symposium on February 23

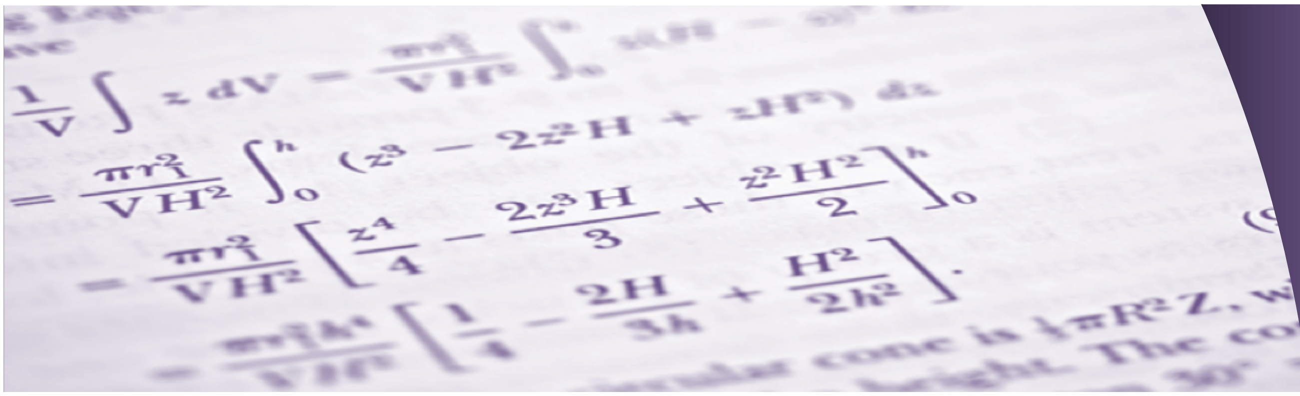
The 15th Annual Texas/Oklahoma Research Undergraduate Symposium (TORUS) will be held at Midwestern State University in Wichita Falls, TX on Saturday, February 19, 2022. Undergraduate students, faculty and any persons interested in the mathematical sciences are encouraged to attend. For more information, contact Dr. Patrick Mitchell at patrick.mitchell@msutexas.edu.

Budapest Semesters in Mathematics

Budapest Semesters in Mathematics Education (BSME) is normally a study abroad program in Budapest, Hungary intended for students interested in the teaching of mathematics at the secondary school level. The summer session will run from June 13 to July 22, 2022.

At BSME, students learn about the Hungarian approach which emphasizes problem solving, mathematical creativity, and communication. The courses are designed so that credits will be transferable to American colleges and universities. BSME is currently accepting applications for Summer 2022. The due date for applications is April 1, 2022, but applications are reviewed on a rolling basis, so students are encouraged to apply early.

More information, including the online application, can be found at https://bsmeducation.com/summer/.



Solution to the November 2021 Problem of the Month

Problem: The behavior of $f(x) = x\sin(1/x)$ near $x = 0$ is a standard example when discussing the Squeeze Theorem or, with $f(0) = 0$ so that $f(x)$ is continuous, differentiability. Is the arclength of the curve y=$f(x)$ between $x =$ 0 and $x = 1$ finite or infinite?

Solution: The arclength is infinite. The arclength is the improper integral

$$\int_0^1 \sqrt{1 + \left[\sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)\right]^2}\, dx.$$

The integrand is always at least 1. We focus on the values of $1/x$ for which cosine is at least 1/2 and sine is negative, namely those for which

$$(2n - 1/3)\pi \le 1/x \le 2n\pi$$

for some positive integer n. The width of this interval is $1/(\pi(12n^2 - 2n))$. The integrand there is greater than

$$\frac{1}{x}\cos\left(\frac{1}{x}\right) \ge \frac{2n\pi - \frac{\pi}{3}}{2} = \left(n - \frac{1}{6}\right)\pi.$$

Thus, the arclength is at least

$$\sum_{n=1}^{\infty}\left(n - \frac{1}{6}\right)\pi \cdot \frac{1}{\pi(12n^2 - 2n)} = \sum_{n=1}^{\infty} \frac{n - 1/6}{12n^2 - 2n},$$

which diverges by the limit comparison test with $\sum_{n=1}^{\infty} 1/n$.

The Problem of Month was solved by Duc Toan Nguyen.

February 2022 Problem of the Month

For all $x \ge y \ge z \ge 0$, show that

$$x^2y + y^2z + z^2x \ge xy^2 + yz^2 + zx^2.$$

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.