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# TCU Math News Letter

Volume 15, Number 1 September 2006

*I met a man once who told me that, far from believing in the square root of minus one, he didn't even believe in minus one.*

-- E. C. Titchmarch (1899 - 1963)

[Editor: Dr. Rhonda Hatcher](#) and [Archive of Newsletters](#)

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## TCU Research Lectureship Series

The Frank Stones TCU Research Lectureship Series for the 2006-2007 academic year will begin on Tuesday, September 12 with a talk by a new member of the TCU Mathematics Department, Professor Greg Friedman. He will present the talk "A gentle introduction to intersection homology." Dr. Friedman's talk will be in Tucker Technology Center 246 at 4:00 p.m.

Professor Paulo Lima-Filho of Texas A&M University will be the second speaker in the Lectureship Series. He will present the talk "Bigraded equivariant cohomology of geometric cellular real varieties" at 4:00 p.m. on Tuesday, September 26 in TTC 226.

All students, faculty, and interested members of the community are invited to attend the Lectureship Series talks. Refreshments will be served before each of these talks in TTC-300 at 3:30 p.m.

## First Parabola Meeting on September 26

The first meeting of Parabola, the TCU undergraduate mathematics student organization, will be on Tuesday, September 26. The meeting will be a pizza lunch at 11:00 a.m. in TTC 300 followed by a talk by Professor Paulo Lima-Filho of Texas A&M University entitled "Geometry in a garden of symmetries."

Parabola talks are open to all students, faculty, and others interested in mathematics. Students interested in joining Parabola should come to a meeting or contact Professor Ken Richardson at [k.richardson@tcu.edu](mailto:k.richardson@tcu.edu).

## Putnam Mathematics Contest

The Sixty-seventh Annual William Lowell Putnam Mathematical Competition will be held on Saturday, December 2, 2006, from 9:00 a.m. to noon and 2:00 to 5:00 p.m. The questions require different levels of mathematical background, and all require a bit of ingenuity to solve. The scores on the exam are typically quite low, and even answering a couple of questions is considered an excellent performance. The competition is open to undergraduates enrolled in colleges and universities of the United States and Canada who have not yet received a college degree.

Those interested in signing up to take the Putnam Exam this year should contact Professor George Gilbert at

[g.gilbert@tcu.edu](mailto:g.gilbert@tcu.edu).

Also, please let Professor Gilbert know if you would like to participate in one or more practice or training sessions over the course of the semester.

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## Solution to the April 2006 Problem of the Month

**Problem:** For what positive real numbers  $p$  is the maximal distance from the point  $(1,0)$  to the curve  $y = x^p$ ,  $0 \leq x \leq 1$ , equal to 1?

**Solution #1 (No calculus):** The maximal distance exceeds 1 if and only if the curve  $y = x^p$  passes above the quarter circle  $(x-1)^2 + y^2 = 1$ ,  $0 \leq x \leq 1$ . This is equivalent to

$$x^p > \sqrt{1 - (x-1)^2} \quad \text{or} \quad x^{2p} + x^2 - 2x > 0$$

for some  $x$  in  $(0,1)$ . For  $p \geq 1/2$ , both  $x^{2p}$  and  $x^2$  are at most  $x$  for all  $x$  in  $[0,1]$ , so  $x^{2p} + x^2 - 2x \leq 0$ .

On the other hand,

$$x^{2p} + x^2 - 2x = x^{2p}(1 + x^{2-2p} - 2x^{1-2p}).$$

For  $p < 1/2$  and  $x$  sufficiently small, it follows that  $1 + x^{2-2p} - 2x^{1-2p} \approx 1 > 0$ , so that the maximal distance exceeds 1.

**Solution #2 (Calculus):** We look at the square of the distance from  $(x, x^p)$  to  $(1,0)$ :  $f(x) = (1-x)^2 + x^{2p}$ . The derivative of  $f$  is  $f'(x) = 2(px^{2p-1} + x - 1)$ . If  $p < 1/2$ , then  $2p-1 < 0$  and  $f'(x) > 0$  for sufficiently small positive values of  $x$  and  $f$  has a maximum greater than  $f(0) = 1$ . On the other hand, for  $p \geq 1/2$ ,  $f(x) \leq (1-x) + x = 1$ .

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## September 2006 Problem of the Month

Are there positive integers  $m$  and  $n$  for which  $m^2 + n^2$  is divisible by  $3(m-n)^2$ ?

**Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.**

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The TCU Math Newsletter will be published each month during the academic year. **Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor.** Items which you would like to have included should be sent to **Dr. Hatcher (Math Dept. Office or P.O. 298900).**