TCU Math News Letter

Volume 4, Number 2 October 1995

The thing that counts is not what we know but the ability to use what we know.

-Leo L. Spears

Editor: Dr. Rhonda Hatcher and Archive of Newsletters

Two Research Lectureship Speakers for October

The TCU Mathematics Department Research Lectureship Series will feature two speakers this month. The first speaker, Professor Ed Cline of the University of Oklahoma, will present a lecture entitled "Modular Representations of Semisimple Group and Quasi-hereditary Algebras" on Tuesday, October 3, 1995.

A second research lectureship talk will be presented on Tuesday, October 24, 1995. The speaker will be Professor Elizabeth Bator of the University of North Texas. She will speak about "Unconditional Convergence in Normed Linear Spaces."

Both of the talks begin at 4 p.m. in Winton Scott Hall. Refreshments will be served in Winton Scott Hall 171 during the half-hour preceding each talk.

Professor Gilbert to Speak at the Next Parabola Meeting

Professor George Gilbert, of the TCU Mathematics Department will present a talk entitled "Fermat's Last Theorem: Special Cases, Polynomials, and History" at the next meeting of Parabola, the TCU Undergraduate Mathematics club. The meeting will be held on Wednesday, October 18, 1995. We will have refreshments in Winton Scott Hall 171 from 3:00-3:30 p.m., and the talk will begin at 3:30 p.m. in Winton Scott Hall 145.

The main subject of Dr. Gilbert's talk is the very famous mathematical result known as Fermat's Last Theorem. This theorem states that the equation

$$x^n + y^n = z^n,$$

where n is an integer greater than 2, has no solution with x, y, and z all nonzero integers. This theorem was first proposed by the French mathematician Pierre de Fermat around 1637. Fermat made this claim in the margin his copy of Diophantus' Arithmetica, and added that he had a proof but that the margin was to small to contain it. The problem remained unsolved for over three hundred and fifty years. It was not until 1995 that a proof was finally completed and published by Andrew Wiles of Princeton University.

Dr. Gilbert will discuss the history of the problem, the special cases n=2 and n=4, the problem with x, y, and z replaced by polynomials, and the extent to which the methods apply to the general problem. This talk should be of interest in students of all levels.

Putnam Exam Sign-up

The deadline for signing up for the Putnam Exam is October 9. Please contact Professor George Gilbert in his office in Winton Scott Hall 141 or by telephone at 921-7335 for more information or to sign up. The exam will be held on Saturday, December 2, 9 a.m. - 12 noon and 2 p.m. - 5 p.m. in Winton Scott Hall. A free pizza lunch will be provided for all participants.

All TCU undergraduate mathematics majors and other undergraduates with an interest in mathematics are encouraged to take the Putnam Exam.

Solution to the September 1995 Problem of the Month

***Problem:** The Fibonacci numbers are defined by* $F_0 = 0$, $F_1 = 1$, *and* $F_{n+1} = F_n + F_{n-1}$. *Show that*

$$\frac{F_{4n}}{F_{2n}} - \left(\frac{F_{2n}}{F_n}\right)^2 = \pm 2.$$

Their solution involves the formula

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}},$$

although a solution directly from the recursion would be interesting.

Solution: Assuming $n > 0$, the formula yields

$$\frac{F_{4n}}{F_{2n}} - \left(\frac{F_{2n}}{F_n}\right)^2 = \frac{\dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^{4n} - \left(\frac{1-\sqrt{5}}{2}\right)^{4n}}{\sqrt{5}}}{\dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^{2n} - \left(\frac{1-\sqrt{5}}{2}\right)^{2n}}{\sqrt{5}}} - \left(\frac{\dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^{2n} - \left(\frac{1-\sqrt{5}}{2}\right)^{2n}}{\sqrt{5}}}{\dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^{n} - \left(\frac{1-\sqrt{5}}{2}\right)^{n}}{\sqrt{5}}}\right)^2.$$

Canceling the $\sqrt{5}$ s and using the difference of squares factorization (!), this simplifies to

$$\frac{F_{4n}}{F_{2n}} - \left(\frac{F_{2n}}{F_n}\right)^2 = \left(\frac{1+\sqrt{5}}{2}\right)^{2n} + \left(\frac{1-\sqrt{5}}{2}\right)^{2n} - \left(\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right)^2$$

$$= -2\left(\frac{1+\sqrt{5}}{2}\right)^n\left(\frac{1-\sqrt{5}}{2}\right)^n = -2(-1)^n = (-1)^{n+1}2.$$

Problem of the Month

Our second problem of the 1995-1996 academic year is to find all pairs of positive integers x and y for which $xy = 2x + 3y + 1995$.

Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

The TCU Math Newsletter will be published each month during the academic year. Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor. Items which you would like to have included should be sent to Dr. Hatcher via e-mail.