TCU MATH NEWSLETTER

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October 2008
Volume 17, Number 2

Man is equally incapable of seeing the nothingness from which he emerges and the infinity in which he is engulfed.

--- Blaise Pascal

TCU Frank Stones Lectureship Research Series Talk on October 28

Professor Amine Fawaz from the University of Texas at Permian Basin will be present the talk ***Volume of meromorphic vector fields on Riemann surfaces*** as part of the Frank Stones Lectureship Research Series. In this talk he will discuss the volume of meromorphic foliations on Riemann surfaces equipped with metrics of constant curvature. He will also give bounds of the volume of foliations given by projectable transversely meromorphic vector fields on principal circle bundles over Riemann surfaces.

The talk will be at 4:00 p.m. on Tuesday, October 28 in Tucker Technology Center 245. Refreshments will be served in TTC 300 at 3:30 p.m. All TCU students and faculty and other interested members of the community are invited to come.

TCU Graduate & Professional School Fair on October 14

Undergraduates interested in attending graduate or professional school after graduation should consider attending the TCU Graduate & Professional School Fair on Tuesday, October 14 in the BLUU Auditorium. The fair is sponsored by TCU Career Services in partnership with Extended Education and the McNair Program. At the Fair, you can meet representatives from graduate and professional programs at TCU and other institutions.

TCU Career Services is also holding free test strategy sessions for the LSAT, GRE, and GMAT. The sessions will be held on October 14 at 3:00 p.m. and at 6:30 p.m. in Sid Richardson Lecture Hall 3. For more information about these programs, call 817-257-2222 or e-mail frogjobs@tcu.edu.

Problems and Solutions

Solution to the September 2008 Problem of the Month

Problem: Let

$$p(x) = x^7 - 3x^6 - 8x^5 - 5x^4 - 7x^3 - 17x^2 - x - 4.$$

Prove that $p(x)$ has exactly one positive root r and that any other real or complex root s satisfies $|s| < r$. (Due to Elgin Johnston.)

Solution: Because $p(0) < 0$ and has positive leading term, it has at least one postiive root. Because it has only one sign change, it can have at most one by Descartes' law of signs. (In fact, a trivial modification of the rest of the proof also shows that there is no second positive root.) For any root s,

The TCU Math Newsletter is published each month during the academic year.

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$$s = 3 + 8/s + 5/s^2 + 7/s^3 + 17/s^4 + 1/s^5 + 4/s^6.$$

When $s \neq r$, $8/s$ is not positive. Now suppose this other root satisfies $|s| \geq r$. Then

$$\begin{aligned} r \leq |s| &= |3 + 8/s + 5/s^2 + 7/s^3 + 17/s^4 + 1/s^5 + 4/s^6| \\ &< 3 + 8/|s| + 5/|s|^2 + 7/|s|^3 + 17/|s|^4 + 1/|s|^5 + 4/|s|^6 \\ &\leq 3 + 8/r + 5/r^2 + 7/r^3 + 17/r^4 + 1/r^5 + 4/r^6 = r, \end{aligned}$$

a contradiction. Therefore, $|s| < r$.

October 2008 Problem of the Month

Two dice each have faces labeled 1, 2, ..., n ($n \geq 4$). Each of the n faces is equally likely to come up when the dice are rolled. What is the probability the sum of the two dice is the same for two rolls of the pair? Your answer should be a simplified quotient of polynomials in n.

Darren Ong's challenge continues. If any TCU student (undergraduate or graduate) submits a correct solution to the Problem of the Month before he does, Darren will dye his "hair bubble-gum pink for at least a week."

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.