TCU Math News Letter

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Reductio ad absurdum, which Euclid loved so much, is one of a mathematician's finest weapons. It is a far finer gambit than any chess gambit: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game.

-- G. H. Hardy

Editor: Dr. Rhonda Hatcher and Archive of Newsletters

Professor Efton Park to Speak at Next Parabola Meeting

The first meeting of Parabola for the spring semester will feature Professor Efton Park of the TCU Mathematics Department.

Dr. Park will present his talk, "Cellular Automata," on Tuesday, February 24, at 3:30 p.m. in Winton Scott Hall 145. Refreshments will be served during the half hour before the talk in Winton Scott Hall 171.

All TCU students, faculty, and other interested members of the community are invited to attend Parabola meetings.

Applications for NSF Research Experiences for Undergraduate Sites Now Being Accepted

The National Science Foundation Research Experiences for Undergraduate Sites for Summer 1998 have been announced. The sites will be at twenty-one different colleges across the United States. A list of the sites along with the email addresses of the site coordinators can be found on the Internet.

Student participants at the REU Sites will be involved in research projects lasting from six to ten weeks in the summer of 1998. The research is done under the direction of faculty. Participants are paid a salary and possibly housing or travel allowances. Last summer, TCU undergraduate Aaron Heap was a student participant at the REU at Trinity University in San Antonio. He found it to be a very positive experience.

Any TCU undergraduate interested in applying for a student participant position at one of the 1998 Sites should check the web site right away since many deadlines are quite soon. If you would like more general information about REU's you can come to discuss them with Professor Hatcher in Winton Scott Hall 142.

TCU Research Lectureship Talks on February 3 and 17

Professor Gerard Walschap of the University of Oklahoma will be next speaker in the TCU Research Lectureship schedule. He will present the talk "Open Manifolds of Nonnegative Curvature" on Tuesday, February 3. A second lectureship talk will be presented by Professor Robert R. Kallman, Distinguished

Research Professor of Mathematics at the University of North Texas on Tuesday, February 17. His talk is entitled "Some Uniqueness Results for Algebraic-Topological Structures."

Both talks begin at 4 p.m. in Winton Scott Hall 145. Refreshments will be served in Winton Scott Hall 171 during the half hour preceding each talk.

Professor Rhonda Hatcher Receives National Mathematics Teaching Award

Professor Rhonda Hatcher of the TCU Mathematics Department was one of three recipients of the 1998 Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching of Mathematics, awarded by the Mathematical Association of America at the Joint Mathematics Meetings in Baltimore, Maryland on January 8, 1998. The other two recipients of the award were Dr. Colin Adams of Williams College and Dr. Rhonda Hughes of Bryn Mawr College.

Solution to the December 1997/January 1998 Problem of the Month Problem

Problem: *How many functions f from $\{1,2,3,4,5,6,7\}$ to itself are there such that the 351-fold composition $f^{(351)}(x) = x$ for every x?*

Solution: There are 351 such functions. Consider the finite sequence i, $f(i)$, $f^{(2)}(i)$, ..., $f^{(7)}(i)$. This sequence must contain at least two identical terms. If $f^{(m)}(i) = f^{(n)}(i)$ with $1 \le m < n$, then

$$f^{(n-1)}(i) = f^{(n-1)}(f^{(351)}(i)) = f^{(350)}(f^{(m)}(i)) = f^{(350)}(f^{(n)}(i))$$
$$= f^{(350)}(f^{(n)}(i)) = f^{(n-1)}(f^{(351)}(i)) = f^{(n-1)}(i).$$

Thus, the first term to reappear in this sequence must be i. Let m_i be the smallest positive integer for which $f^{(m_i)}(i) = i$. (Note that $m_i \le 7$.) This implies $f^{(rm_i)}(i) = i$ for positive integers r, so that $f^{(n)}(i)$ depends only on the remainder when n is divided by m_i. The hypothesis $f^{(351)}(x) = x$ implies that 351 is divisible by m_i. Factoring $351 = 3^3 \cdot 13$, we see that $m_i = 1$ or 3 for each i. Thus, we may group $\{1,2,3,4,5,6,7\}$ into single elements with $f(i) = i$ and triples (i,j,k) with $f(i) = j$, $f(j) = k$, and $f(k) = i$. We can choose one single element and two sets of triples in $\binom{7}{1}\binom{6}{3}/2 = 70$ ways. Each set of triples $\{i,j,k\}$ allows two choices of f: $f(i) = j$, $f(j) = k$, $f(k) = i$ and $f(i) = k$, $f(k) = j$, $f(j) = i$. There are 280 such f. Similarly there are $\binom{7}{4}2 = 70$ functions with 4 single elements and 1 triple, and 1 with 7 single elements (the identity). In all, this totals 351 elements. (Had $f^{(3n)}(x) = x$ with n relatively prime to $2 \cdot 5 \cdot 7 = 70$, there are 351 functions as well.)

Problem of the Month

This month's problem by Kenneth M. Wilkie was first published in a 1980 issue of The Pentagon. In a prison sits a prisoner who is sentenced to die. Fortunately the warden, an eccentric, offers the prisoner a chance to live. The warden gives the prisoner 12 black balls and 12 white balls. Next the warden gives the

prisoner two boxes and tells the prisoner that tomorrow the executioner will draw one ball at random from one of the boxes. If a white ball is drawn, the prisoner will be freed; if a black ball is drawn the sentence will be carried out. How should the prisoner arrange the balls in the boxes so as to maximize his chance for freedom?

Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

The TCU Math Newsletter will be published each month during the academic year. Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor. Items which you would like to have included should be sent to Dr. Hatcher (Math Dept. Office or P.O. 298900).