TCU MATH NEWSLETTER

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Mathematics is an independent world created out of pure intelligence.

--- William Wordsworth (1770 -1850)

Mathematics Talks at TCU in February

Three visiting professors will be presenting talks at TCU during the month of February. All students are strongly encouraged to attend. The early talks, given at 1 or 2 p.m., are specifically targeted to students.

Our first speaker will be Professor Qiao Zhang of Johns Hopkins University. Dr. Zhang will present two talks on Monday, February 11. His student talk, entitled ***Dining, Internet and Number Theory***, will be at 1 p.m. in Winton Scott Hall 170. He will present his research talk, ***Mean Values of L-functions***, at 4 p.m. in TTC 138.

Professor Sandy Spiroff of Seattle University will be our second speaker in February. Her student talk is entitled ***Unique Factorization and the Roll of the Dice***, and she will present it at 1 p.m. on Friday, February 15 in TTC 138. A pizza lunch for students will be served in TTC 300 before the student talk. Her research talk, ***Class Group Maps on Hypersurfaces from a Commutative Algebra Perspective, with Connections to Algebraic Geometry and K-theory***, will be in the same room that day at 4 p.m.

The third speaker in February will be Professor Brent Doran from the Institute for Advanced Study in Princeton, New Jersey. His talks will be on February 18. Please watch the TCU Mathematics Department web page for the details.

TCU Career Expo on February 13

The *TCU Career Expo* for Spring 2008 will be held in the Campus Recreation Gym on Wednesday, February 13 from 4 to 7 p.m. Over ninety employers will be represented. Full-time jobs, part-time jobs, and internships will be available. All TCU students and alumni are welcome to attend. Students attending should dress professionally and brings résumés.

To make the most of the Career Expo, students are encouraged to attend one of the Career Expo Prep Workshops. They are being offered at 4 p.m. on February 6 in SC 202, at noon on February 11 in SC 202, at 4 p.m. on Feburary 12 in DRH 134, and on February 13 at 10 a.m. in SC 202.

iJunior is a portion of Career Expo especially for the Class of 2009. Juniors can win prizes, search for internships, order class rings, and learn about graduate school.

For more information and a list of employers planning to attend go to www.frogjobs.net.

The Math Newsletter Now Online Only

The *TCU Math Newsletter* will now be posted on the TCU Mathematics Department website only, rather than sent through the mail on paper. New issues will be announced via e-mails sent to all TCU mathematics majors, MAT students, math faculty, and other interested readers. The newsletter will be posted on the TCU Mathematics Department web site at www.math.tcu.edu/Newsletters/Archive.html.

If you are not receiving an e-mail announcement of new issues of the *TCU Math Newsletter* and but would like to, please send your e-mail address to r.hatcher@tcu.edu so that you will be on our e-mail list.

Problems and Solutions

Solution to the November 2007 Problem of the Month

Problem: Starting from $x = 0$, a fair coin is flipped n times. Each time a head occurs, 1 is added to x; after each tail, 1 is subtracted from x. Let a_n be the average (mean) of the absolute value of x after n flips. Does $a_n \to \infty$ as $n \to \infty$?

Solution: Yes, $a_n \to \infty$ as $n \to \infty$.

When $x \neq 0$, we next obtain either x-1 or x+1, each with probability 1/2. Thus, the average absolute value does not change. When $x = 0$, the average absolute value increases from 0 to 1. The probability that $x = 0$ after n flips is 0 for n odd and ${}_nC_{n/2} \,/\, 2^n$ for n even. Therefore, a_{n+1} equals a_n for n odd and $a_n + {}_nC_{n/2} \,/\, 2^n$ for n even. It follows that

$$a_{2n+2} = a_{2n+1} = \sum_{k=0}^{n} \frac{{}_{2k}C_k}{2^{2k}}$$

and that

$$\lim_{n\to\infty} a_n = \sum_{k=0}^{\infty} \frac{{}_{2k}C_k}{2^{2k}} .$$

Now ${}_{2k}C_k$ is the largest of the $2k$+1 binomial coefficients ${}_{2k}C_j$, $j = 0,1, \ldots, 2k$. Since these coefficients sum to 2^{2k}, we have

$$\frac{{}_{2k}C_k}{2^{2k}} > \frac{1}{2k+1} .$$

Thus a_n diverges to ∞ by comparison with $\sum 1 \,/\, (2k+1)$.

Remark: In fact, for $k > 1$,

$$\frac{{}_{2k}C_k}{2^{2k}} = \frac{(2k)!}{(2\cdot4\cdots(2k))^2} = \frac{1\cdot3\cdots(2k\text{-}1)}{2\cdot4\cdots(2k)} = \frac{1}{\sqrt{2}}\cdot\frac{3}{\sqrt{2\cdot4}}\cdot\frac{5}{\sqrt{4\cdot6}}\cdots\frac{2k\text{-}1}{\sqrt{(2k\text{-}2)\cdot(2k)}}\cdot\frac{1}{\sqrt{2k}}$$

$$> \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2k}} = \frac{1}{2\sqrt{k}} .$$

and

$$\frac{{}_{2k}C_k}{2^{2k}} = \frac{\sqrt{1\cdot3}}{2}\cdot\frac{\sqrt{3\cdot5}}{4}\cdots\frac{\sqrt{(2k\text{-}1)\cdot(2k+1)}}{2k}\cdot\frac{1}{\sqrt{2k+1}} < \frac{1}{\sqrt{2k+1}} .$$

Still more precise asymptotic estimates can be obtained through Stirling's approximation.

February 2008 Problem of the Month

Show that there is no (everywhere) increasing cubic polynomial passing through the points (0,0), (1,1), and (2,16).

Darren Ong continues to offer his challenge. If any TCU student (undergraduate or graduate) submits a correct solution to the Problem of the Month before he does, Darren will dye his "hair bubble-gum pink for at least a week."

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

The TCU Math Newsletter is published each month during the academic year.

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