TCU Math Newsletter

Volume 2, Number 4 December 1993 - January 1994

It requires a very unusual mind to make
an analysis of the obvious.

– Alfred North Whitehead

Math Department Holiday Buffet on December 14

The TCU Mathematics department will hold its annual Holiday Buffet from 11:00 a.m. to 1:00 p.m. on Tuesday, December 14 in Winton Scott Hall 171. As always, we will have a great spread of food, including turkey, dressing, and an endless supply of sweets. All TCU mathematics majors and graders are invited to come.

If you would like to come, please come to the Math Department office to sign up to bring an example of your cooking, or students can just pay $1.00. We hope you can join us and enjoy a nice break during final exam week.

Parabola Talk on January 25

Professor Rhonda Hatcher will present the first Parabola Talk for the Spring 1994 semester. The talk is entitled "Additive Number Theory." It requires no special mathematical background, and it should be accessible to all undergraduates. The talk will be given on Tuesday, January 25 at 3:30 p.m. in Winton Scott Hall 145, and refreshments will be served at 3:00 p.m. in Winton Scott Hall 171.

All TCU students are invited to attend meetings of Parabola, whether or not you are a member. If you are interested in joining Parabola, see Dr. Rhonda Hatcher.

Graders and Clinic Workers Needed For Spring Semester

If you are interested in working for the Mathematics Department as a grader or Math Clinic worker for the Spring 1994 semester you should let Professor Charlie Deeter know before leaving for the holiday break, and then see him again as early as possible after returning in January (preferably before classes begin). We are especially interested in finding a Math Clinic worker who can help students in Elementary Statistics (Math 1043). Dr. Deeter's office is in Winton Scott Hall 159.

Problem of the Month

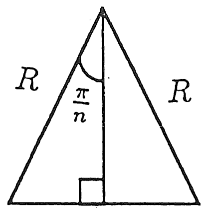
Little Caesar's Pizza is currently advertising a special where the customer can get up to 5 toppings on each of two medium pizzas. They state that there are 1,048,576 possible combinations for an order. How many toppings are available at Little Caesar's?

Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 32903). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

Solution to the November 1993 Problem of the Month

Problem: *Can two noncongruent regular polygons have the same area and the same perimeter?*

Solution:



No. A regular polygon with n sides can be inscribed in a circle of radius R for some R. The polygon can be divided into n identical triangles like the one pictured. The area of this triangle is $R^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n}$, and the length of the base is $2R \sin \frac{\pi}{n}$. Thus, the area $A(n, R)$ and perimeter $P(n, R)$ are given by the following formulas:

$$A(n,R) = nR^2 \sin \tfrac{\pi}{n} \cos \tfrac{\pi}{n}$$
$$P(n,R) = 2nR \sin \tfrac{\pi}{n}$$

It is clear from these formulas that two n–gons with the same area or perimeter also have the same radius R (and are thus congruent). Now, suppose that $A = A(n,R) = A(m,Q)$ and $P = P(n,R) = P(m,Q)$. Then $A = nR^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} = mQ^2 \sin \frac{\pi}{m} \cos \frac{\pi}{m}$, and $P = 2nR \sin \frac{\pi}{n} = 2mQ \sin \frac{\pi}{m}$. We can write

$$\frac{P^2}{4A} = \frac{n \sin \frac{\pi}{n}}{\cos \frac{\pi}{n}} = \frac{m \sin \frac{\pi}{m}}{\cos \frac{\pi}{m}}, \text{ so that}$$

$$n \tan \tfrac{\pi}{n} = m \tan \tfrac{\pi}{m}, \quad \text{or} \quad \frac{\tan \frac{\pi}{n}}{\pi/n} = \frac{\tan \frac{\pi}{m}}{\pi/m}$$

Now, notice that the function $f(x) = \frac{\tan x}{x}$ is strictly increasing on the interval $(0, \frac{\pi}{2})$. (Observe that $\frac{\tan x}{x}$ is the slope of the line between the point $(0,0)$ and the point $(x, \tan x)$ on the graph of $y = \tan x$. This slope is clearly increasing on this interval.) Thus, equality can hold only if $m = n$.

The TCU Math Newsletter will be published each month during the academic year.
Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor.
Items which you would like to have included should be sent to Dr. Hatcher (Math Dept. Office or P.O. 32903).