# **TCU Math News Letter**

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 $F\!\!\begin{pmatrix}a_1,\ldots,a_{\rm eff}\\b_1,\ldots,b_{\rm ff}\end{pmatrix}\!\!=\sum_{k\in 0}\frac{a_1^k\cdots a_{\rm eff}^k}{b_1^k\cdots b_{\rm ff}^k}\frac{z^k}{k!}$ 

Anything that has survived for centuries with such awesome notation must really be useful.

-- Ronald L. Graham, AT&T Laboratories Donald E. Knuth, Stanford University Oren Patashnik, Stanford University

Editor: Dr. Rhonda Hatcher and Archive of Newsletters

# **Problem Solving Seminar**

Professor George Gilbert plans to run a problem solving seminar next fall, provided that at least three students are interested. If you are interested in problem solving, you can participate either informally (no credit, free) or as a one-hour, Math 4970, Special Topics, course. The emphasis will be on gaining experience with contest-type problems and on writing clear, complete solutions. Both aspects will be of benefit to you in upper level mathematics courses. Contact Dr. Gilbert (Winton Scott 141, 921-7335, g.gilbert@tcu.edu) right away if you might be interested. This is highly recommended for those who would like to take the Putnam mathematics contest in December.

### **Calculus Bee Winners**

Mitsutaka Shirasaki, a junior engineering and mathematics major, took first place in the TCU Calculus Bee, which was held on Monday, March 31. Second place went to sophomore mathematics major Jeff Moles, and Aaron Heap, a junior mathematics major placed third. Thirty-five students participated in the Bee. Participants were eliminated after missing three questions. It took 21 questions and over two hours to determine the winners this year.

### **Parabola Meeting on April 8**

Professor Pamela Marcum of the TCU Physics Department will be the speaker at the next meeting of Parabola on Thursday, April 10. Her talk is entitled "Mathematical Techniques in Observational Astronomy."

The Parabola meeting will begin with refreshments at 3:30 p.m. in Winton Scott Hall 112, and Professor Marcum's talk will begin at 4:00 in Winton Scott Hall 145. All TCU students, faculty, and other members of community are invited to attend.

# **TCU Lectureship Series**

The TCU Mathematics Department Research Lectureship Series will feature two more speakers in the 1996-1997 academic year. The next speaker will be Dr. Ron Douglas, Provost of Texas A & M University. His talk, "Operator Theory and Algebraic Geometry," will be presented on Wednesday, April 30 at 4 p.m. The last speaker this year will be Professor Wieslaw Zelazko of the Math Institute of the Polish Academy of Science in Warsaw, Poland. His talk, the title of which will be announced later, will be at 4 p.m. on Friday, May 2.

All of the TCU Lectureship talks are presented in Winton Scott Hall 145. Refreshments are served in the half-hour preceding each talk in Winton Scott Hall 112.

## Mathematics Department Picnic on Saturday, May 3

The Mathematics Department Picnic, sponsored by Parabola, the TCU Undergraduate Mathematics Club, will begin at 1 p.m. on Saturday, May 3 at the home of Dr. Rhonda Hatcher and Dr. George Gilbert at 4204 Harlanwood Drive. Their home is only about one and a half miles from the TCU campus. All TCU students, faculty, and friends are invited to come. A sign-up sheet and maps to the picnic are in the Math Department Office in WSH 112.

## Solution to the March 1997 Problem of the Month

Problem: A goat is tied to the corner of a 10 foot square enclosure by a 30 foot rope. The goat is able to graze anywhere outside the enclosure that the 30-foot rope permits. What is the area that the goat is able to graze in square feet?

Solution: Set up a coordinate system so that the goat is at the origin and the adjacent vertices of the square are at (10,0) and (0,10), as in the diagram below.



The grazing area consists of three-fourths of a circle of radius 30, which has area  $675\pi$ , two sectors of a circle of radius 20, and two triangular regions, one of which is ABC. The point C is on the line y = x at a distance 20 from (10,0). The distance formula implies  $(x-10)^2 + y^2 = (x-10)^2 + x^2 = 20^2$ , which leads to C =  $(5\sqrt{7}, 5\sqrt{7})$ . We then see that sin  $\angle CAD = (\sqrt{7} + 1)/4$ , so that the area of sector ACD is 200 arcsin( $(\sqrt{7} + 1)/4$ ). The altitude of triangle ABC from C to AB is  $5\sqrt{7} - 5$ , so triangle ABC has area  $25\sqrt{7} - 25$ . Combining results, the area grazed by the goat is

$$675\pi + 400 \arcsin((\sqrt{7} + 1)/4) + 50\sqrt{7} - 50 \approx 2661.5687.$$

This month's problem was correctly solved by students Vanessa Fielding and Amy Ludington. Another method of solution leads to the answer

$$875\pi - 400 \arcsin((\sqrt{7} - 1)/4) + 50\sqrt{7} - 50$$

from which it follows that

$$\arcsin((\sqrt{7} + 1)/4) + \arcsin((\sqrt{7} - 1)/4) = \pi/2.$$

Can you prove this identity?

#### **Problem of the Month**

Raffle tickets for a \$500 prize sell for \$1. If you are the only person left who may buy tickets and 200 have already been sold to others, how many tickets should you buy in order to have the largest possible average profit?

Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

The TCU Math Newsletter will be published each month during the academic year. Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor. Items which you would like to have included should be sent to Dr. Hatcher (Math Dept. Office or P.O. 298900).